Ordinary Differential Equations

Math 677-001

Fall 2018

This is the web page **http://math.cos.gmu.edu/~wanner/courses/m677f18/index.html** It will be updated regularly and always contain the latest information on the course. This website is only for general policies concerning the course, as well as for the continuously updated syllabus. For all other information on the course, including homework and the scanned lecture notes, please go to Blackboard.

General Information:

Instructor:	Thomas Wanner		
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E-mail:	twanner@gmu.edu		
Web Page:	http://math.cos.gmu.edu/~wanner/		
Fax:	(703) 993-1491		
Office hours:	M 2-3pm, W 3-4pm, and by appointment		

Lectures:	MW 4:30-5:45pm, Exploratory Hall 4106
-	A thorough knowledge of advanced calculus and linear algebra is assumed. Also, familiarity with the theorem-proof style of presentation is important.
	Differential Equations and Dynamical Systems, Third Edition, by Lawrence Perko (Springer, 2006).

Important Links:

- <u>Detailed syllabus</u> (including a list of recommended books)
- Relevant official GMU policies

Syllabus:

This course provides an introduction to the dynamics of ordinary differential equations. Specific topics include existence theory, linear and nonlinear systems of ordinary differential equations, stability theory of equilibrium solutions, invariant manifolds, and elementary bifurcation theory. It will also be demonstrated how these methods can be used in applications. A more detailed syllabus can be found <u>here</u>. It will be updated weekly.

Homework Assignments:

Homework problems will be assigned once a week and posted on Blackboard. Some of these assignments will be graded and count towards your homework score. While the remaining ones do not have to be handed in, I do advise everyone strongly to study them and write out the solutions properly. I will go through many of the homework problems in the following class and you will not benefit from this if you have not made a serious attempt at solving them.

Grading Policy:

Your final grade in the course will be determined from graded homework assignments, your performance in a midterm exam, and a comprehensive final exam (dates and details to be announced). Weights for these items will be distributed approximately according to the following schedule:

Homework	Midterm Exam	Final Exam	Attendance
40%	25%	25%	10%

Thomas Wanner, August 17, 2018.

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The following table contains a preliminary schedule for the course. This page will be updated regularly throughout the semester.

Week Date

I. Introduction

Remarks

- 1 08/27 1. What is an Ordinary Differential Equation?
 - 2. Examples from Mechanics and Biology
 - 08/29 3. Basic Definitions and Results

2 09/03 No class! (Labor Day)

09/05 II. Existence Theory

- 1. The Fundamental Existence and Uniqueness Theorem
- 2. Global Existence of Solutions
- 3. Dependence on Initial Conditions and Parameters
- 4. Autonomous Differential Equations and Dynamical Systems
- 5. Rescaling of Vector Fields

III. Linear Systems

- 1. Homogeneous Systems
- 2. Inhomogeneous Systems
- 3. Exponentials of Matrices
- 4. Computation of Matrix Exponentials
- 5. Planar Linear Systems
- 6. Stability Theory

IV. Local Theory of Nonlinear Systems

- 1. Linearized Stability
- 2. The Effect of Forcing
- 3. Invariant Manifolds and Sets
- 4. Global Invariant Manifolds
- 5. Stable and Unstable Manifolds
- 6. Center Manifolds
- 7. Approximation of Invariant Manifolds
- 8. The Hartman-Grobman Theorem
- 9. Elementary Bifurcations

V. Global Theory of Nonlinear Systems

1. Limit Sets and Attractors

2. Lyapunov Functions

11/21 No class! (Thanksgiving Break)

- 3. Gradient and Hamiltonian Systems
- 4. Poincare-Bendixson Theory
- 5. Limit Cycles
- 6. Wazewski Principle
- 7. A Taste of Conley Index

16 12/12 Final Exam (4:30-7:15pm)

For supplementary reading, I recommend the following books:

- Herbert Amann: Ordinary Differential Equations, Walter de Gruyter, 1990.
- Carmen Chicone: Ordinary Differential Equations with Applications, Second Edition, Springer, 2006.
- John Guckenheimer and Philip Holmes: Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer, 1990.
- Yuri A. Kuznetsov: Elements of Applied Bifurcation Theory, Third Edition, Springer, 2004.
- Lawrence Perko: Differential Equations and Dynamical Systems, Third Edition, Springer, 2006.
- Steven H. Strogatz: Nonlinear Dynamics and Chaos, Westview Press, 2001.

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Relevant George Mason Official University Policies

The following policies apply to all courses at George Mason University:

- 1. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
- 2. You are responsible for knowing the last days to drop and add this class.
- 3. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
- 4. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot change be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
- 5. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
- 6. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
- 7. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.